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THEORETICAL FOUNDATIONS OF THE PARTICLE SPECTRUM

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Abstract

A relation between the radii and masses of the quarks and leptons is established for each fermion generation. Kinematical calculations provide a proof of the essential stability of the proton at rest within the standard model together with conditions for the reaction $p \rightarrow uuu + e^- + \bar{\nu}_e$ with a uuu composite state as a byproduct. The Koide relation reduces the scaling to a single number, and a theoretical explanation is given in terms of graded symmetries. The values of the masses of the each of the quarks are derived through nonperturbative infinite-genus effects.

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1. Introduction

There have been four major results in the production of the masses in the particle spectrum. The integer multiple rule is phenomenological law stating that many of the resonances have masses that are integer multiples of the pion [10] It can be phrased in terms of pion bubbles in the self-enery diagrams. The linear Regge trajectories in the graph of the angular momentum versus the squared mass [12] eventually led to the discrovery of string theory, It may be conjectured that the linearity in both relations may be related, with the energy of the pions being converted into increases in the angular momentum and squared mass. The systematic formalism developed for the theory of the strong interactions introduced an SU(3) symmetry group with the hadron spectroscopy based on the quark model [6]. Finally, an empirical relation between the masses of the leptons in different generations [7] had been found, and it reflected an underlying discrete component of the automorphism group of the spinor space of the standard model, S_3 [5].

Therefore, the remaining problems in connection with the particle spectrum are the ratios of the quark and lepton masses, the origin of the scaling factor between the generations of leptons, the interpretation of the neutrino and its ratio of its mass to that of the lepton, and the derivation of the quark masses. Each of these topics will be considered in this investigation, and the solutions are described within a nonperturbative model.

The theoretical problem of the η' mass can be solved only partially either through a quadrature formula consisting of the singlet and octet masses of the pseudoscalar multiplet, a pseudovector model of bound states of quarks and anti-quarks or the introduction of a Goldstone boson. None of these techniques yields the exact value of the η' mass equal to 957.78 MeV/c^2. The first two configurations have a mass of approximately 831.08 MeV/c^2. The remainder cannot be predicted from the multiplet structure or perturbation theory, and therefore, it may be attributed to nonperturbative effects. Given the equality of the η' wavefunction with $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$. When the nonperturbative effects are identified with the ends of infinite-genus surfaces, its contribution is sufficient to generate the difference between the η' mass and the initial estimate. It will be established further that the boundary components of the geometrical model of the u and d quarks is sufficient to produce the masses in the particle spectrum. Since the s quark will be contructed from the chiral u quark, the electron and these boundary components, the s quark then does not have an effect on the nonperturbative component of the η' mass. Then, it may be established that the u and u quarks have an infinite-genus components with masses of 61.85 MeV/c^2 and 64.85 MeV/c^2. The inverse proportionality of the self-energy of a charged particle with no composite structure and its radius yields a formula for the radii of the quarks. The derived value for the radii of the chiral up and down quarks is demonstrated in §2 to compatible with. a conventional nearly point-particle model and the distance between quarks in the nucleon required for mass difference between the proton and the Δ^{++} baryon.

The mass scalings of the leptons in the three fermion generations depends their string representation together with the Koide relation. A theoretical explanation of the scaling coefficients is described in §3 based on massive representations of a superalgebra, and a generalization of this supersymmetry to a $\mathbb{Z}_4$ grading. Whereas the radii of the masses of the charged muon and τ lepton may be derived from m_e, from the self energy and the inverse proportionality with respect to the mass, the description of the neutrinos neutrinos in terms of linear extent of the string requires a proportionality between the mass of the neutrino and the radius. Then it follows that the size of the neutrino is considerably less, which is consistent with its almost pointlike structure.

The second and third generation quarks are much heavier and characterized by new quantum numbers such as strangeness. The entire hadronic spectrum may be represented by multiplets in the three dimensions of isospin, hypercharge and strangeness. Therefore, it is not unexpected that there exists a model of these quarks and baryons constructed from the first generation quarks, anti-quarks and the nonperturbative boundary components. The group theoretical transformations and the masses will place conditions on the complex of the u, $\bar{u}$, d and $\bar{d}$ quarks. It is proven in §4 that the masses of s, c, b and t quarks is equal to a set of sums of the masses of the chiral u and d quarks and the infinite-genus ends. The number of first generation quarks and anti-quarks increases with the mass, and the radii of the heavier quarks becomes larger. The hadronic spectrum is established then through the same formula in terms of the quarks.

This phenomenological model requires a proof of the mass of the chiral quarks and the occurrence of the ends of surfaces of infinite genus in the string description of strongly interacting particles. One method for deriving the chiral mass would consists of developing the perturbative expansion of low energy effective theories of the hadronic interactions. The nonpertubative effect may be identified with the infinite-genus term in a generalization of the series expansion in string theory of a correlators of products of field strengths and the duals. The mass would have to be related to the potential energy of boundaries of these surfaces.

2. Radii of the Quarks in the First Fermion Generation and Proton Stability

Quarks have charge, isospin and colour, and therefore, the dynamics is governed by the electromagnetic, weak and strong nuclear forces at atomic scales. The contribution of the charge to the mass is known to be

$$(1)\qquad m_{elec} = \frac{e^2}{6\pi\epsilon_0 r c^2}$$

$$E_{elec}^{self-energy} = m_{elec}c^2 = \frac{3}{4}m_{tot}c^2$$

when there are only gravitational and electromagnetic interactions. This formula yields an inverse proportionality between the ratios of the masses and the ratios of the radii of particles acting only as a result of these two forces. Suppose, in the first instance, that this condition is valid for the electron and the up quark. An analysis of the mass of the η' meson demonstrates that the perturbative mass, even with a vector component and a summation over quark annihilation diagrams, equals 831.08 MeV/c^2. Since the experimental value is 957.78 MeV/c^2, the remainder may be attributed to nonperturbative effects including an infinite-genus component that equals 126.6 MeV/c^2. This difference can be split into the the sum of a u quark contribution of 61.85 MeV/c^2 and a d quark contribution of 64.85 MeV/c^2, which can be deduced from the $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ sector of the boundary component, with mass $\frac{1}{2}(m_{u\bar{u}\infty} + m_{d\bar{d}\infty}) = \frac{1}{2}(2m_{u\infty} + 2m_{d\infty}) = m_{u\infty} + m_{d\infty}$, Both quarks would have the same mass from chiral perturbation theory, $m_{uc} = m_{dc} = 243.15 MeV/c^2$. Since the formula for the electromagnetic part of the mass requires a compact spherical model of the particle, it is this mass that will be substituted Then

$$(2)\qquad \frac{m_e}{m_{uc}} = \frac{e^2}{\left(\frac{2}{3}e\right)^2}\frac{r_{uc}^{(0)}}{r_e}$$

where m_{uc} is the mass of the quark derived from breaking of chiral invariance, such that the nonperturative contribution equals nearly half of the difference between the perturbative mass of the η' meson and its experimental value [4], $r_{uc}^{(0)}$ is the leading-order estimate of the quark radius. Given that the radius of the electron is $3 \times 10^{-16}m$,

$$(3)\qquad r_{qc}^{(0)} = \frac{4}{9}\frac{0.511\ MeV/c^2}{243.15\ MeV/c^2} \times 3 \times 10^{-16}m = 2.8021458298 \times 10^{-19}m,$$

which is closer to the experimental value than $2.1979 \times 10^{-19}m$ predicted for a quark mass of 310‘MeV/c^2. There is a correction to this estimate from the linear term in the strong interaction potential $V(r) = 1GeV/fm \times r$. Over the diameter of the quark, the accumulated potential energy is

$$(4) \qquad 1\ GeV/fm \times 2(2.8021458298 \times 10^{-19}m) = 0.56042916596\ MeV$$

With the inclusion of this energy, the mass must equal that given by the chiral gap. The value required in the Lorentz formula must be shifted to $242.58957087404\ MeV/c^2$. Substituting this value into the formula for r_{uc} gives

$$(5) \qquad r_{uc} = 2.808563687 \times 10^{-19}m.$$

The effect of the weak interaction is negligible and this value for the radius can be compared with experiment. The radius of the electron is indicative of a prototype of the proton consisting of an electron and two positrons, which have barely enough space inside a nucleon. Then, the modern version of the proton formed by replacing these constituents by two up quarks and one down quark, which may be achieved by transferring a charge of $\frac{1}{3}e$ from each positron to the electron and placing it in an SU(3) gauge field to transform these particles to quarks with fractional charges. By contrast, the neutron cannot be formed by three charges of magnitude e. Instead, a fourth electron would be necessary to make the composite neutral and must combine with one of the up quarks in inverse beta decay $\bar{\nu}_e + e^- + p \rightarrow n$ and $\bar{\nu}_e + e^- + u \rightarrow d$ and, after the radiation of the neutron, the standard composition of the neutron in terms of an up quark and two down quarks is present. Beta decay $n \rightarrow p + e^- + \bar{\nu}_e$ causes the neutron to have lifetime of approximately 18 minutes.

The proton is known to be essentially stable at rest with a lifetime longer than 10^{32} years [11]. Grand unified theories have been developed with a superselection rule to prevent reactions with the mediation of leptoquarks or scalar fields, with the potential $p \rightarrow e^+\pi^0$ and other decays [3] allowed instead. The decay of the down quark in the proton, generating $uud \rightarrow uuu + e^- + \bar{\nu}_e$, nevertheless, may be considered. Suppose that the distance between the each pair of quarks is d_{qq}. Since

$$(6) \qquad E_{elec}(uud) = \frac{4}{9}\frac{e^2}{4\pi\epsilon_0 d_{qq}} - \frac{2}{9}\frac{e^2}{4\pi\epsilon_0 d_{qq}} - \frac{2}{9}\frac{e^2}{4\pi\epsilon_0 d_{qq}} = 0,$$

the location of equidistant quarks in the proton does not effect the binding energy. However, now let the d quark decay to a u quark, electron and an anti-eletrcon neutrino at the fixed location in the baryon. A reaction $d \rightarrow u + W^- \rightarrow u + e^- + \bar{\nu}_e$, together with $p \rightarrow (uuu) + e^- + \bar{\nu}_e$ may be considered with $m_{(uuu) \approx m_p}$ initially. By energy and momentum conservation

$$(7) \qquad \begin{aligned} E_d &= E_u + E_{W^-} \\ \vec{p}_d &= \vec{p}_u + \vec{p}_{W^-} \end{aligned}$$

A d quark can decay into an u quark and a W^- boson with a non-zero scattering angle. The momentum transfer may be sufficient to deflect the entire uuu state simultaneously in the same direction. Otherwise, this composite state would disintegrate with the third u quark moving away from the other two quarks. In the latter circumstatnce, the alignment of the momentum vectors would require either the W^- boson to move in the same or the opposite direction as the u quark. The conservation laws then become either

$$\gamma_d m_d c^2 = \gamma_u m_u c^2 + \gamma_{W^-} m_{W^-} c^2 \qquad (8)$$
$$\gamma_d m_d v_d = \gamma_u m_u v_u + \gamma_{W^-} m_{W^-} v_{W^-} = 0$$

and

$$\frac{\gamma_{W^-}(v_d - v_{W^-})}{\gamma_u(v_u - v_d)} = \frac{m_u}{m_{W^-}} \qquad (9)$$

when the W^- boson moves in the same direction as the u quark or

$$\gamma_d m_d c^2 = \gamma_u m_u c^2 + \gamma_{W^-} m_{W^-} c^2 \qquad (10)$$
$$\gamma_d m_d v_d = \gamma_u m_u v_u - \gamma_{W^-} m_{W^-} v_{W^-}$$

and

$$\frac{\gamma_{W^-}(v_d + v_{W^-})}{\gamma_u(v_u - v_d)} \frac{m_u}{m_{W^-}} \qquad (11)$$

if the W^- boson moves in the opposite direction to the u quarks. Under both conditions, there exists a range of velocities $v_d,\ v_u \gg v_{W^-}$ and $v_d < v_u < \left[\left(1 - \frac{m_u^2}{m_d^2}\right) c^2 + \frac{m_u^2}{m_d^2} v_d^2\right]^{\frac{1}{2}}$ which include solutions to the above relations.

Therefore, even under the constraint of the alignment of momentum vectors in the reaction $p \rightarrow (uuu) + W^- \rightarrow (uuu) + e^- + \bar{\nu}^e$, the kinematics allow the transformation to the new composite state. The influx of a positive electostatic potential energy generates an increase in the mass of the (uuu) state.

$$\frac{2.0395145184 \times 10^{-9} eVm}{d_{dd}}. \qquad (12)$$

Given that $m_{\Delta^{++}} = 1272\ MeV/c^2$, equality of the shift in the electrostatic potential energy with the difference in the rest mass energies

$$m_{\Delta^{++}} c^2 - m_p c^2 = 333.5 MeV \qquad (13)$$

requires

$$E_{elec}(uuu) = \frac{4}{9} \frac{e^2}{4\pi\epsilon_0 d_{qq}} \times 3 = \frac{4}{3} \frac{e^2}{4\pi\epsilon d_{qq}} = \frac{2.0395145184 \times 10^{-9} eVm}{d_{dd}}. \qquad (14)$$

and

$$d_{qq} = 6.115485 \times 10^{-18} m. \qquad (15)$$

If the only force at this distance is electostatic in origin, ther would be nothing preventing the u quarks moving radially outwards and the composite state disintegrating. Consequently, the onset of the strong interactions is necessary to restore the constitution of the eventual (uuu) state, which then as a mass increased by 333.5‘MeV/c^2. Equilibrium would be reached at a distance greater than or equal to $6.115485 \times 10^{-18} m$.

The kinematical conditions also may be given when the byproduct has a greater rest mass. The Δ^{++} baryon is both heavier than the proton and has angular momentum $\frac{3}{2}$. Given the decay $d \to u + W^- \to u + e^- + \bar{\nu}_e$, the reaction $p \to \Delta^{++} + W^-$ may be considered at non-zero momentum since charge, isospin and angular momentum are conserved. Again, the momentum vector of Δ^{++} wil be selected to be aligned along the direction of the motion of the proton, and similarly, $\vec{p}_{W^-} \parallel \vec{p}$. By the conservation of energy and momentum,

$$\begin{aligned} E_p &= E_{\Delta^{++}} + E_{W^-} \\ \vec{p}_p &= \vec{p}_{\Delta^{++}} + \vec{p}_{W^-}. \end{aligned} \tag{16}$$

Suppose that $\vec{p}_{\Delta^{++}}$ and $\vec{p}_{W^-}$ are pointing in the same direction. Then

$$|\vec{p}_p| = |\vec{p}_{\Delta^{++}}| + |\vec{p}_{W^-}| \tag{17}$$

From the two equations

$$\begin{aligned} \gamma_p m_p c^2 &= \gamma_{\Delta^{++}} m_{\Delta^{++}} c^2 + \gamma_{W^-} m_{W^-} c^2 \\ \gamma_p m_p v_p &= \gamma_{\Delta^{++}} m_{\Delta^{++}} v_{\Delta^{++}} + \gamma_{W^-} m_{W^-} v_{W^-}, \end{aligned} \tag{18}$$

$$\gamma_{\Delta^{++}} m_{\Delta^{++}} (v_p - v_{\Delta^{++}}) + \gamma_{W^-} m_{W^-} (v_p - v_{W^-}) = 0. \tag{19}$$

Since $m_p < m_{\Delta^{++}}, m_{W^-}$, $v_p > v_{\Delta^{++}}, v_{W^-}$. The above relation produces a contradiction because $v_p - v_{\Delta^{++}} > 0$ and $v_p v_{W^-} > 0$.

Now suppose that $\vec{p}_{\Delta^{++}}$ and $\vec{p}_{W^-}$ point in opposite directions. It follows that

$$|\vec{p}_p| = |\vec{p}_{\Delta^{++}}| - |\vec{p}_{W^-}|. \tag{20}$$

and

$$\gamma_p m_p v_p = \gamma_{\Delta^{++}} m_{\Delta^{++}} v_{\Delta^{++}} - \gamma_{W^-} m_{W^-} v_{W^-}. \tag{21}$$

Combining this equation with the energy conservation relation gives

$$\gamma_{\Delta^{++}} m_{\Delta^{++}} (v_p - v_{\Delta^{++}}) + \gamma_{W^-} m_{W^-} (v_p + v_{W^-}) = 0. \tag{22}$$

A contradiction results if $v_p - v_{\Delta^{++}} > 0$. Let $v_{\Delta^{++}} > v_p$. Then $\gamma_{\Delta^{++}} > \gamma_p$ and

$$\gamma_{\Delta^{++}} m_{\Delta^{++}} + \gamma_{W^-} m_{W^-} > \gamma_p m_p, \tag{23}$$

which contradicts the conservation of energy. Therefore, the reaction $p \to \Delta^{++} + W^{-}$ cannot occur when the momentum vectors are aligned. These contradictions can be circumvented by decreasing the components of the momentum along the direction of the d quark motion and allowing the projection along of the u quark and W boson momenta along the perpendicular to cancel.

3. Scaling of the Masses of the Leptons

It is evident from beta decay that the down quark must contain an electron and the radius would be much larger than that of the up quark and approximately equal to $3 \times 10^{-16}m$. The difference in the masses of the up and down quarks may be attributed to nonperturbative effects [4]. The electron and the anti-neutrino then can be traced to characteristics of infinite-genus surfaces. Since the anti-neutrino is customarily regarded as a point particle, the identification with a point source of non-zero harmonic measure on the ideal boundary is consistent.

The relative size of the electron and neutrino may be described. Point sources on the ideal boundary represent ends of the Riemann surface, and with a boundary of the end being diffeomorphic to a circle, the electron and neutrino shall be viewed as circular strings. A spherical model of these particles would be generated from a rapid revolution of the string about its axis. With a uniform linear density, the greater extent yields the larger mass. By contrast with the Lorentz formula, the mass increases with the size of the circular arc.

The proportionality of the ratios of radii with the ratios of the masses yields

$$r_{\nu_e} = \frac{m_{\nu_e}}{m_e} r_e. \tag{24}$$

The electron neutrino mass is measured to be $0.007614\ eV$ and

$$\frac{m_e}{m_{\nu_e}} = 6.71132 \times 10^{7}. \tag{25}$$

It follows that

$$r_{\nu_e} = \frac{3 \times 10^{-16} m}{6.71132 \times 10^{7}} = 4.47 \times 10^{-24} m. \tag{26}$$

There is a scaling of the masses of the leptons from the first to second generations and second to third generations. Let $\sqrt{\frac{m_\mu}{m_e}} = a$ and $\sqrt{\frac{m_\tau}{m_e}} = b$. By the Koide relation [7],

$$1 + a^2 + b^2 = \frac{2}{3}(1 + a + b)^2. \tag{27}$$

A rearrangement of terms gives the quadratic equation

$$b^2 - 4(1+a)b + (1 - 4a + a^2) = 0. \tag{28}$$

Of the two solutions to this equation,

$$2(1+a) \pm \sqrt{3}(a+2), \tag{29}$$

the lesser root is excluded because $m_\tau > m_\mu$. The larger root is

$$b = (2+\sqrt{3})a + 2(1+\sqrt{3}). \tag{30}$$

The values $a \approx 14.5$, $b \approx 59.578$ generate the experimental charged lepton mass ratios.

A theoretical basis for the mass scales of the leptons in the fermion generations requires an explanation of the relation between the supercharges and the number 14.5. The example of a massive representation of N = 2 Poincare superalgebra

$$\begin{aligned} \{Q^a_\alpha, Q_{\dot{\alpha}b}\} &= 2\sigma^\mu{}_{\alpha\dot{\alpha}} P_\mu \delta^a{}_b \\ \{Q^a_\alpha, Q^b_\beta\} &= 2^{\frac{3}{2}} \epsilon_{\alpha\beta}\epsilon^{ab} Z \\ \{Q^\dagger_{\dot{\alpha}a}, Q^\dagger_{\dot{\beta}b}\} &= 2^{\frac{3}{2}} \epsilon_{\dot{\alpha}\dot{\beta}}\epsilon_{ab} Z \end{aligned} \tag{31}$$

furnishes the anticommutators

$$\begin{aligned} \{a_\alpha, a^\dagger_\beta\} &= \delta_{\alpha\beta}(M + \sqrt{2}Z) \\ \{b_\alpha, b^\dagger_\beta\} &= \delta_{\alpha\beta}(M - \sqrt{2}Z), \end{aligned} \tag{32}$$

where

$$\begin{aligned} a_\alpha &= \frac{1}{2}\left(Q^1_\alpha + \epsilon_{\alpha\beta}(Q^2_\beta)^\dagger\right) \\ b_\alpha &= \frac{1}{2}\left(Q^1_\alpha - \epsilon_{\alpha\beta}(Q^2_\beta)^\dagger\right). \end{aligned} \tag{33}$$

The central charge then can decrease the trace of the anticommutators of supercharges. For example, when the number of spinor indices is increased from 2 to 4 in an N = 4 superalgebra, tracing of a set of anticommutators can give a factor of $16(M - 3\sqrt{2}Z)$. Setting the parameter Z of internal rotations commuting with the supersymmetry generators equal to $\frac{1}{48}M$, the factor becomes $(16 - \sqrt{2})M$.

The ratio $\sqrt{\frac{m_\mu}{m_e}}$ can be set equal to $16 - \sqrt{2}$ only if the supercharges rather than the masses are scaled by this factor. Therefore, a $\mathbb{Z}_4$ graded symmetry that squares to a supersymmetry is necessary. One of the anticommutation relations of the $\mathbb{Z}_4$ graded algebra would have the form

$$\{Q_{\frac{1}{2}}{}^{a}{}_{\tilde{\alpha}\tilde{\gamma}}, Q^{b\,\dagger}_{\frac{1}{2}}{}_{\tilde{\gamma}\tilde{\beta}}\} = (\gamma^{\mu})_{\tilde{\alpha}\tilde{\gamma}}(Q_{\mu})_{\tilde{\gamma}\tilde{\beta}}\delta^{a}{}_{b}, \tag{34}$$

where two indices $(\alpha\tilde{\alpha})$ correspond to a single four-component index. Since $\tilde{\alpha}\tilde{\gamma}$ can cover only four indices, the elements of $(\gamma^{\mu})_{\tilde{\alpha}\tilde{\gamma}}$ belong to the diagonal. Then $(\gamma^{\mu})_{\tilde{\alpha}\tilde{\gamma}}(Q_{\mu})_{\tilde{\gamma}\tilde{\beta}}$ can be written as $(\gamma^{\mu})_{\alpha\alpha}(Q_{\mu})_{\beta}\delta^{\alpha\beta}$, where repeated indices are summed and Q_0 is set equal to the supercharge Q. The scales of the supercharges may be arranged to increase by $16-\sqrt{2}$ from the first to the second generation, while there would be an overall scaling of $(16-\sqrt{2})^2$ for the masses.

The ratio of the lifetimes of the muon and τ lepton is known to be a constant multiple of the ratio of the fifth powers of the masses. By contrast, the Lorentz formula requires the ratio of the radii to be proportional to the inverse of the ratio of the masses. It follows that

$$\begin{aligned} r_{\mu} &= \frac{m_e}{m_{\mu}} r_e = 1.4509 \times 10^{-18} m \\ r_{\tau} &= \frac{m_e}{m_{\tau}} r_e = 8.628 \times 10^{-20} m \end{aligned} \tag{35}$$

Corrections resulting from the weak interactions may be deduced from the width of the resonances $\Gamma = \frac{G_F^2 E_0^5}{60\pi^3}$. Setting E_0 equal to $m_{\mu}c^2 = 105.658\ MeV$ or $m_{\tau}c^2 = 1777\ MeV$, the shifts in the mass are found to be

$$\begin{aligned} \Delta m_{\mu} &= 10^{-14} MeV/c^2 \\ \Delta m_{\tau} &= 1.61 \times 10^{-8} MeV/c^2 \end{aligned} \tag{36}$$

and the effect on the masses and radii is negligible.

There have been various figures given for the masses of the neutrinos for each flavour and the mass eigenstates [8]. A comparison with the Koide relation is compatible with a normal hierarchy. The neutrino mass matrix [1] requires a mixing angle [2] larger than the Cabibbo angle. Generally, the masses derived from the mass matrix and the eigenstates yield an inverted hierarchy with values of $m_{\nu_{\mu}}$ and $m_{\nu_{\tau}}$ which are generally not sufficiently large [9]

4. Quarks in the Second and Third Fermion Generations

The mass of the strange quark exceeds that of two $\bar{u}$ anti-quarks in chiral perturbation theory by less than 1 MeV/c^2, when the value $m_{\bar{u}c} = 243.15\ MeV/c^2$ is used. Combining two $\bar{u}$ anti-quarks with a positron produces a composite system of charge $-\frac{1}{3}e$ with mass

$$2m_{\bar{u}c} + m_{e^+} = 486.811\ MeV^2. \tag{37}$$

Beginning with a dd state, a charge of $\frac{1}{3}e$ would have to be added without introducing a $\bar{d}$ anti-quark. Instead, contact with a $\bar{d}$ anti-quark must result in the transfer of the charge $\frac{1}{3}e$ and a recoil of the anti-quark for this configuration to model a particle of charge $-\frac{1}{3}e$ and mass nearly equal to m_s.

Both composite systems, however, do not satisfy a mandatory condition on the representation theory of SU(3)$_c$. Quarks transform under the **3** representation and anti-quarks belong to the $\overline{\mathbf{3}}$ representation of SU(3). Since the positron is an SU(3) singlet state, it is not tenable for $\bar{u}\bar{u}e^+$ to transform under this group identically to the strange quark s. Similarly, the $dd^{(\frac{1}{3})}$ state transforms under the $\mathbf{3} \times \mathbf{3} = \mathbf{8} \oplus \mathbf{1}$ representation, which is also different.

An alternative configuration consistent with representation theory of $SU(3)$ would be the a ue^- state together with four-infinite genus components with a mass

(38)

$$\begin{aligned} m_{uc} + m_{e^-} + 4m_{u\infty} &= 243.15\ MeV/c^2 + 0.511\ MeV/c^2 + 4(61.85\ MeV/c^2) \\ &= 491.061\ MeV/c^2. \end{aligned}$$

The sum of the radii of the chiral up quark and the boundary of the infinite-genus components equals

$$r_{uc} + 4r_{u\infty} = r_{uc} + 4\frac{61.85}{0.511}r_{e^-} = 1.45244618676 \times 10^{-13}m \tag{39}$$

which is much too large for the strange quark to fit inside a baryon from the decuplet that includes the Ω^-. Instead, the boundary of the infinite-genus surface must be wrapped around the fundamental constituent up quark. Its final radius is measured to be $5.95268 \times 10^{-19}m$. Setting

$$2\pi \sum_{n=0}^{N} (2.808563687 \times 10^{-19}m + n\Delta x) = 2\pi(1.45244618676 \times 10^{-13}m), \tag{40}$$

it follows that

$$\begin{aligned} N &= 3.31562 \times 10^5 \\ \Delta x &= 9.4827406 \times 10^{-25}m. \end{aligned} \tag{41}$$

The electrostatic potential energy between the u quark and the electron

$$V_{ue^-}^{elec} = -\frac{2}{3}\frac{e^2}{4\pi\epsilon_0 d_{ue^-}} = -\frac{1.0705775916 \times 10^{-9}eVm}{d_{ue^-}}. \tag{42}$$

must be included in the computation. When this electroatic energy is discharged into the bound state during the process of fusion at a distance of

$d_{ue^-} = 3\times 10^{-16} m$, $V^{elec}_{ue^-} = 3.598591972\ MeV$ yielding $m_s = 491.061\ MeV/c^2 - 3.598591972\ MeV/c^2 = 487.462408028\ MeV/c^2$ which belongs to the range of experimental error.

The charmed quark may be described as a complex consisting of dd, u and $\bar{d}\bar{d}$, with one u, two d and two $\bar{d}$ infinite-genus components. Then the mass would be

$$(43)\qquad \begin{aligned} m_c &= m_{uc} + 2m_{dc} + 2m_{\bar{d}c} + m_{u\infty} + 2m_{d\infty} + 2m_{\bar{d}\infty} \\ &= 5(243.15\ MeV/c^2) + 4(64.85\ MeV/c^2) + 61.85\ MeV/c^2 \\ &= 1543\ MeV/c^2 \end{aligned}$$

which is consistent with the mass of the J/ψ particle. The electrostatic potential energy of the complex describing the c quarks is $V^{c}_{elec} = 2V^{ud}_{elec} + 2V^{u\bar{d}}_{elec} + V^{dd}_{elec} + V^{\bar{d}\bar{d}}_{elec} + 2V^{d\bar{d}hor.}_{elec} + V^{d\bar{d}diag.}_{elec}$. When the u quark is placed centrally in a square with d and $\bar{d}$ quarks at the corners, $V^{ud}_{elec} + V^{u\bar{d}}_{elec} = 0$ and $V^{dd}_{elec} + V^{\bar{d}\bar{d}}_{elec} + 2V_{d\bar{d}hor.} = 0$. The remainder is

$$(44)\qquad 2V^{d\bar{d}diag.}_{elec} = -\frac{2}{9}\frac{e^2}{4\pi\epsilon_0 d_{d\bar{d}diag.}} = -\frac{3.598591972\times 10^{-10} eVm}{d_{d\bar{d}diag.}}.$$

When the distance between the d and $\bar{d}$ quarks across the diagonal of the square array is $8.3688185395 \times 10^{-18} m$, the mass of the charmed quark is reduced to $1.5\ GeV/c^2$.

The quarks in the third fermion generation require many more components. The Υ resonance at an energy of $9.46\ GeV$ is a 1^- $b\bar{b}$ state if $m_b \approx 4.73\ GeV/c^2$. The following configuration

$$\begin{array}{ccccccccc} u & & \bar{u} & u & & \bar{u} & u & & \bar{u} \\ & d & & & \bar{d} & & & d & \\ u & & \bar{u} & u & & \bar{u} & u & & \bar{u} \end{array}$$

has a mass equal to

$$(45)\qquad \begin{aligned} &6m_{uc} + 6m_{\bar{u}c} + 2m_{dc} + m_{\bar{d}c} + 6m_{u\infty} + 6m_{\bar{u}\infty} + 2m_{d\infty} + m_{\bar{d}\infty} \\ &= 4584\ MeV/c^2 \end{aligned}$$

The difference from the experimental value of the b quark mass can be explained through the addition of infinite-genus components neighbouring quarks and anti-quarks and a reduction as a result of the electrostatic potential energy. If the d and $\bar{d}$ quarks are placed symmetrically between the uu and $\bar{u}\bar{u}$ pairs, the electrostatic energies involving the u and $\bar{u}$ quarks will cancel with the exception of neighbouring the diagonal interaction between u and $\bar{u}$ quarks. When the planar array is replaced by a connected three-dimensional configuration, there is a negative electrostatic potential energy between the $\bar{u}\bar{u}$ in the third complex and the uu pair in the first complex. If the three complex are arranged in an equilateral triangular form, the distances between the central d and $\bar{d}$ quarks will become equal.

Then the electrostatic potential energy of the entire supercomplex would be

$$12V_{u\bar{u}} + V_{dd} + V_{d\bar{d}} + 2V_{\bar{d}d} + V_{dd} + V_{d\bar{d}} = 12V_{u\bar{u}} + 2V_{\bar{d}d} \tag{46}$$

These electrostatic energies equal

$$\begin{aligned} V_{elec}^{u\bar{u}diag.} &= -\frac{4}{9}\frac{e^2}{4\pi\epsilon_0 d_{u\bar{u}diag.}} = -\frac{7.197183944 \times 10^{-10} eVm}{d_{u\bar{u}diag.}} \\ V_{elec}^{d\bar{d}} &= \frac{1}{9}\frac{e^2}{4\pi\epsilon_0 d_{d\bar{d}}} = -\frac{1.799295986 \times 10^{-10} eVm}{d_{d\bar{d}}}. \end{aligned} \tag{47}$$

The length of the diagonal from the u to $\bar{u}$ quark is $\sqrt{2}$ times that of the length of side of a single complex. The distance between the d and $\bar{d}$ quarks would equal the length of this side.

When the distance between the surfaces of the d and $\bar{d}$ quarks is $6.335412 \times 10^{-18} m$,

$$\begin{aligned} V_{elec}^{u\bar{u}} &= -80.3290105\ MeV \\ V_{elec}^{d\bar{d}} &= -28.4\ MeV \end{aligned} \tag{48}$$

The net electrostatic potential energy is

$$12V_{u\bar{u}} + 2V_{d\bar{d}} = -1020.748126\ MeV. \tag{49}$$

The strong interaction potential would contribute significantly to the energy unless it drops rapidly to zero beyond a range between $10^{-18}m$ and $10^{-17}m$. Since $6m_{u\infty} + 6m_{\bar{u}\infty} + 2m_{d\infty} + m_{\bar{d}\infty} = 12(61.85\ MeV/c^2) + 3(64.85\ MeV/c^2) = 936.75\ MeV/c^2$, the mass of the entire supercomplex of 15 elements is

$$\begin{aligned} &4584\ MeV/c^2 + 936.75\ MeV/c^2 - 1020.748126\ MeV/c^2 \\ &= 4500.001874\ MeV/c^2. \end{aligned} \tag{50}$$

It may be noted that the sum of the masses of the supercomplexes and the boundary components gives $4584\ MeV/c^2 + 936.75\ MeV/c^2$. Therefore, the Υ resonance would be created by a fusion of the b and $\bar{b}$ quarks with the electrostatic potential energy of a single complex approximately equal to $-184.81\overline{66}\ MeV$, which can be achieved by increasing the distances between the d and $\bar{d}$ quarks to $8.1781346 \times 10^{-18}m$.

The top quark is much heavier with a mass of 175 GeV/c^2. Let n be number of complexes of the form

$$\begin{array}{ccccccc} d & & \bar{d} & & d & & \bar{d} \\ & u & & or & & \bar{u} & \\ d & & \bar{d} & & d & & \bar{d} \end{array}$$

Each complex has a chiral mass of $5(243.15\ MeV/c^2) = 1215.75\ MeV/c^2$. Given that there are infinite-genus components of surfaces emanating from each of the chiral quarks, the entire mass contributed by n complexes

(51)

$$n(1215.75\ MeV/c^2)+4n(64.85\ MeV/c^2)+n(61.85\ MeV/c^2) = n(1537\ MeV/c^2).$$

Setting n equal to 113, it would equal 173.681 GeV/c^2. Given that there are two other infinite-genus ends originating from the central 113 u and $\bar{u}$ quarks, their connections generate another

$$226(61.85\ MeV/c^2) = 13.9781\ GeV/c^2. \tag{52}$$

Adding the two masses gives an estimate in excess of m_t by 12.6591 GeV/c^2. The remainder of the supercomplex would close to form an electrically neutral structure.

There is a reduction in energy resulting from the electrostatic forces between the a three-dimensional polygonal configuration with alternating central u and $\bar{u}$ quarks. With 57 u quarks and 56 $\bar{u}$ quarks in the complex, the electrostatic potential energies amongst the central quarks and anti-quarks cancel except for $56 \times 2\ V_{elec}^{u\bar{u}}$. The diagonal interactions yield $113 \times 4\ V_{elec}^{d\bar{d}}$.

Setting $d_{u\bar{u}} = 1.09104255722 \times 10^{-17} m$ and $d_{d\bar{d}} = 1.54296713803 \times 10^{-17} m$,

$$\begin{aligned} V_{elec}^{u\bar{u}} &= -65.9661146814\ MeV \\ V_{elec}^{d\bar{d}} &= -11.6612722439\ MeV \end{aligned} \tag{53}$$

The mass is reduced by 12.6590998986 GeV to $m_t \doteq 175\ GeV/c^2$.

5. **Conclusion**

The mass spectrum of hadrons has been successfully described through the quark model and the spin coupling formula. Further regularities in the integer multiple rule, Regge trajectories and the Koide relation have resulted in substantial understanding of the elementary particle interactions. The patterns in the fermion multiplets of the standard model have yet to be entirely elucidated. The quarks are found to have a mass generated by chiral perturbation theory and a nonperturbative portion that may be attributed to the infinite-genus term in a string theoretic version of a series expansion of expectation values of products of field strengths. Many of the conventional methods may be adapted to the chiral mass of the up quark, which is 243.15 MeV/c^2. This value is sufficient to establish an inverse proportionality of the ratio of the masses of the up quark and the electron to the ratio of the radii by the Lorentz formula. A similar calculation yields the radii of the muon and τ lepton. Since the beta decay requires the reaction $d \to u + e^- + \bar{\nu}_e$, and the difference between the masses of the up and down quarks is a nonperturbative effect, the electron and the anti-neutrino are identified with characteristics of the end of an infinite-genus surface. By constrast with the Lorentz formula, the extent of the boundary circle representing the two particles is proportional to the mass, given a uniform linear density. Then the radius of the anti-neutrino is computed to be eight orders of magnitude less than that of the electron. The analogue of the beta decay for the proton, converting a uud to a uuu state, is found to be kinematically impossible when its momentum is zero or aligned with the momentum vectors of the byproducts in the intermeidate channel as a result of the larger rest mass of the Δ^{++} baryon. Therefore, it is verified that a proton at rest is stable within the standard model, and a similar conclusion is valid when it has momentum unless a composite *uuu* state of lesser rest mass is created or the momentum transfer allows a non-zero scattering angle.

The Koide relation for the masses of the charged leptons has a theoretical explanation in terms of an anticommutation relation for the supercharges and an approximation for disparate masses. Defininining two variables a and b equal to $\frac{\sqrt{m_\mu}}{\sqrt{m_e}}$ and $\frac{\sqrt{m_\tau}}{\sqrt{m_e}}$ respectively, a quadratic equation for b in terms of a may be derived, with the solution $=(2+\sqrt{3})a+2(1+[\sqrt{3})$. The derivation of the scaling of the first generation, a, requires the introduction of a coefficient in the trace of the anticommutators of the form $16 - \sqrt{2}$, which is possible with central charges in the algebra. A consideration of four-dimensional superalgebras reveals that this factor can be introduced for masses. However, consistency with the scaling a requires that it is present for supercharges, which requires a $\mathbb{Z}_4$ graded supersymmetry. The specific form of each of the anticommutation relations, and the central charge, would be a necessary for this theoretical explanation to be valid.

Given the infinite-genus components representing the nonperturbative contributions to the u and d masses, various models may be constructed for the quarks in the second and third fermion generations. Group theory of the $SU(3)_c$ makes it mandatory that the configurations transform under the **3** representation. An equivalence between the strange quark and a combination of the up quark and the electron can be completed when four-infinite-genus components are added, the electron is scattered and the ideal boundary is wrapped around the chiral quark. Complexes of up and down quarks and the anti-quarks with five elements yield a composite system that satisfies this condition, with the correct net charge. Then a sequence of identical copies of the complex and the antiparticle version would comprise a much larger structure which models the more massive quarks. The central quarks are attached to two infinite-genus components in the third generation, and there is a reduction resulting from the electrostatic potential energy. By this technique, the c, b and t quarks can be described by 1, 3 and 113 complexes. This sequence must be explained for a rigorous theory of the structures of the quarks.

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